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Closed and open FRW cosmologies with matter creation: Kinematic tests

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Abstract. The limits imposed by the classical cosmological tests on closed and open FRW universes driven by adiabatic matter creation are investigated. Exact expressions for the lookback time, age of the universe, luminosity distance, angular diameter, and galaxy number counts versus redshift are derived and their meaning discussed in detail. An interesting consequence of these cosmological models is the possibility of an accelerated expansion today (as indicated from supernovae observations) with no need to invoke either a cosmological constant or an exotic “quintessence” component.

Key words: Cosmology: theory

1. Introduction

It is widely known that the standard cold dark matter FRW cosmology present serious theoretical and observational difficulties to be considered an acceptable description of the Universe. An overlook in the literature shows the existence of a growing body of work discussing alternative cosmologies (Carvalho et al. 1992, Krauss and Turner 1995, Lima et al. 1996, Caldwell et al. 1997, Overduin and Copperstock 1998). The first motivation comes from the conflict between the age of the universe (which is proportional to the Hubble parameter), and the age of the oldest stars in globular clusters. The ages of the globular clusters typically fall upon the interval $t_{gc} = (12 - 14)\text{Gyr}$ (Bolton and Hogan 1995, Pont et al. 1998, Riess et al. 1999), while measurements of the expansion time scale of the Universe are now converging to $h = (H_o/100\text{km/sec/Mpc}) = 0.7 \pm 0.1$ (Freedman 1998). For this value of the “little” h , the theoretically favoured Einstein-de Sitter Universe predicts an age of the Universe ($t_o = \frac{2}{3}H_o^{-1}$) within the interval $8.1\text{Gyr} \leq t_o \leq 10.8\text{Gyr}$. For a generic FRW cosmology, the generality of this

problem comes from the fact that t_o is always smaller than H_o^{-1} . Indeed, the “age conflict” is even more acute if we consider its variant based on the age constraints from old galaxies at high redshift (Dunlop 1996, Krauss 1997). As recently argued, the overall tendency is that if more and more old redshift galaxies are discovered, the relevant statistical studies in connection with the “age problem” may provide very restrictive constraints for any realistic cosmological model (Alcaniz and Lima 1999).

Another important piece of data is provided by the recent measurements of the deceleration parameter from SNe Ia observations. Using approximately fifty type Ia supernovae, with redshifts between 0 and 1, two groups have presented strong evidence that the universe may be accelerating today ($q_o < 0$). This result is in apparent contradiction with a universe filled only by nonrelativistic matter, in such a way that even open models or more generally, any model with positive deceleration parameter seems to be in disagreement with these data (Perlmutter et al. 1998, Riess et al. 1999).

These problems inspired several cosmologists to consider models with a second relic component (an exotic kind of matter, probably of nonbaryonic origin) which is seen only by its gravitational effects (Krauss and Turner 1995, Turner and Write 1997, Chiba et al. 1997). Among these scenarios, considerable attention has been dedicated to models with a cosmological constant Λ , a primeval scalar field (Ratra and Peebles 1988), decaying vacuum cosmologies (Overduin and Copperstock 1998), as well as a noninteracting x -component (Silveira and Waga 1997, Caldwell et al. 1998).

On the other hand, scenarios with a different kind of ingredient, namely, an adiabatic matter creation process, has also been proposed in the literature (Lima et al. 1996, Lima and Abramo 1999). The limits imposed by the classical cosmological tests on a class of dust filled flat FRW cosmologies with matter creation have also been examined (Lima and Alcaniz 1999, hereafter paper I). In this sort of cosmology, the age of the universe may be

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large enough to agree with the observations, and more important still, there is no need to invoke a second smooth component in order to generate a negative deceleration parameter.

In this context, the aim of the present work is to extend the treatment of the paper I to include both the elliptic ($k = +1$) and hyperbolic ($k = -1$) Universes. The paper is organized as follows. Next section we set up the basic equations for FRW type cosmologies endowed with an adiabatic matter creation process. The classical cosmological tests are described and compared with the flat case in section 3.

2. Universes with Adiabatic Creation : Basic Equations

We start with the homogeneous and isotropic FRW line element ($c = 1$)

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) , \quad (1)$$

where r , θ , and ϕ are dimensionless comoving coordinates, $k = 0, \pm 1$ is the curvature parameter of the spatial sections and $R(t)$ is the scale factor.

In models with “adiabatic” creation, the dynamic behavior is determined by the Einstein field equations (EFE) together the balance equation for the particle number density (Prigogine et al. 1989, Calvão et al. 1992, Lima and Germano 1992)

$$8\pi G\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} , \quad (2)$$

$$8\pi G(p + p_c) = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} , \quad (3)$$

$$\frac{\dot{n}}{n} + 3\frac{\dot{R}}{R} = \frac{\psi}{n} , \quad (4)$$

where an overdot means time derivative and ρ , p , n and ψ are the energy density, thermostatic pressure, particle number density and matter creation rate, respectively. The creation pressure p_c depends on the matter creation rate, and for “adiabatic” creation, it takes the following form (Calvão et al. 1992, Lima and Germano 1992)

$$p_c = -\frac{\rho + p}{3nH}\psi , \quad (5)$$

where $H = \dot{R}/R$ is the Hubble parameter.

To give a complete description, the set (2-5) must be supplemented by an equation of state, which in the cosmological domain is usually given by

$$p = (\gamma - 1)\rho \quad (6)$$

where the γ parameter specifies if the universe is radiation ($\gamma = \frac{4}{3}$) or dust ($\gamma = 1$) dominated.

Now, according to paper I (see also Lima et al. 1996), we assume that the matter creation rate is

$$\psi = 3\beta nH , \quad (7)$$

where the β parameter must be determined either from a kinetic theoretical approach or from quantum field theory in curved spacetime. In general, the β parameter is a function of the cosmic era, or equivalently, of the γ parameter. Assuming that only the creation of the dominant component contributes appreciably to the matter content, we should have at least two parameters, β_r and β_m , for each phase of the Universe (radiation and matter). However, since we are particularly interested in the present matter dominated phase, from now on we take $\gamma = 1$ and $\beta_m = \beta$ supposed to be constant and defined on the interval $[0, 1]$.

Combining equations (2), (3), (4), 5 and (7), is readily seen that the evolution equation for the scale factor reads

$$R\ddot{R} + \Delta\dot{R}^2 + \Delta k = 0 , \quad (8)$$

the first integral of which is

$$\dot{R}^2 = \frac{A}{R^{2\Delta}} - k , \quad (9)$$

where $\Delta = \frac{3(1-\beta)-2}{2}$. By expressing the constant A in terms of the present day parameters (see Eq.(2)), it is straightforward to show that the above equation can be written as

$$\left(\frac{\dot{R}}{R_o}\right)^2 = H_o^2 \left[1 - \Omega_o + \Omega_o \left(\frac{R_o}{R}\right)^{1-3\beta} \right] . \quad (10)$$

where $\Omega_o = \frac{\rho}{\rho_c}|_{t=t_o}$ and $H_o = \frac{\dot{R}}{R}|_{t=t_o}$ are the present values of the density and Hubble parameters. For $\beta = 0$ the above equation reproduces the standard cold dark matter FRW result (Kolb and Turner 1990). In virtue of the matter creation, we also see that the explicit dependence of the energy density on the scale factor $R(t)$ is slightly modified in comparison with the standard case. Combining (2) and (9) one finds

$$\rho = \rho_o \left(\frac{R_o}{R}\right)^{3(1-\beta)} , \quad (11)$$

where $\rho_o = 3A/8\pi GR_o^{3(1-\beta)}$. It thus follows from the definition of Δ , that all the expressions of physical interest are obtained from the standard ones simply by replacing the “index” $\gamma = 1$ by an effective parameter $\gamma_{eff} = 1 - \beta$. This explains why a dust dominated Universe may have a dynamic behavior equivalent to a Universe filled with a matter component with negative pressure.

Following standard lines we also define the deceleration parameter $q_o = -\frac{R\ddot{R}}{\dot{R}^2}|_{t=t_o}$. Using equations (2), (6) and (7) one may show that

$$q_o = \frac{1 - 3\beta}{2}\Omega_o . \quad (12)$$

Therefore, for any value of $\Omega_o \neq 0$, we see that the deceleration parameter q_o with matter creation is always smaller than the corresponding one of the FRW model. The critical case ($\beta = \frac{1}{3}, q_o = 0$), describes a “coasting cosmology”. However, instead of being supported by “K-matter” (Kolb 1989), this kind of model is obtained in the present context for a dust filled universe, and the corresponding solutions hold regardless of the value of Ω_o . It is also interesting that even negative values of q_o are allowed for a dust filled Universe, since the constraint $q_o < 0$ can always be satisfied provided $\beta > 1/3$. These results are in line with recent measurements of the deceleration parameter q_o using Type Ia supernovae (Perlmutter et al. 1998; Riess et al. 1999). Such observations indicate that the universe may be accelerating today, which corresponds dynamically to a negative pressure term in the EFE. For a fixed Ω_o , this means that the universe with creation is older than the corresponding FRW model with the usual deceleration parameter $q_o \geq 0$. This behavior also reconcile other recent results (Freedman 1998), pointing to a Hubble parameter H_o larger than 50 km s⁻¹ Mpc⁻¹. To date, only scalar field models (Ratra and Peebles 1988), and the so-called “quintessence” (of which Λ is a special case) have been invoked as being capable of explaining these results (Caldwell et al. 1998). As remarked before, in the present framework, is the creation pressure that provides the additional acceleration measured by a negative q_o , and not an exotic equation of state as in models dominated either by the cosmological constant or a “quintessence” ($\gamma < 1$) (Caldwell et al. 1998, Huey et al. 1998).

3. Kinematic Tests

The kinematical relation distances must be confronted with the observations in order to put limits on the free parameters of the model. Now, we derive the kinematical relations for the closed and open cases of the model considered here. The limits imposed by these tests will be compared with the ones imposed on the flat case (paper I) in the last section.

a) Lookback time-redshift Diagram

The lookback time, $\Delta t = t_o - t(z)$, is the difference between the age of the Universe at the present time ($z = 0$) and the age of the Universe when a particular light ray at redshift z was emitted. By integrating (10) such a quantity is easily derived

$$t_o - t(z) = H_o^{-1} \int_{(1+z)^{-1}}^1 [1 - \Omega_o + \Omega_o x^{-(1-3\beta)}]^{-1/2} dx \quad (13)$$

which generalizes the standard FRW result (Kolb and Turner 1989). The age of the Universe is obtained by taking the limit $z \rightarrow \infty$ in the above equation. We find

$$t_o = H_o^{-1} \int_0^1 [1 - \Omega_o + \Omega_o x^{-(1-3\beta)}]^{-1/2} dx \quad (14)$$

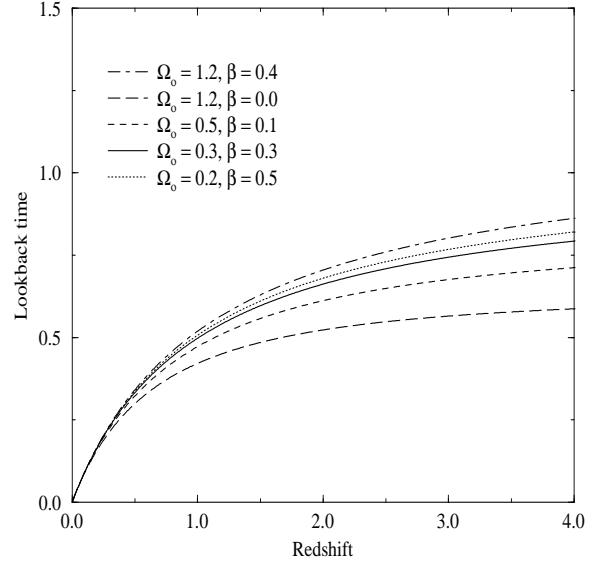


Fig. 1. Lookback time as a function of the redshift for some selected values of Ω_o and β . The lookback time increases for higher values of β , i.e., models with larger matter creation rate are older.

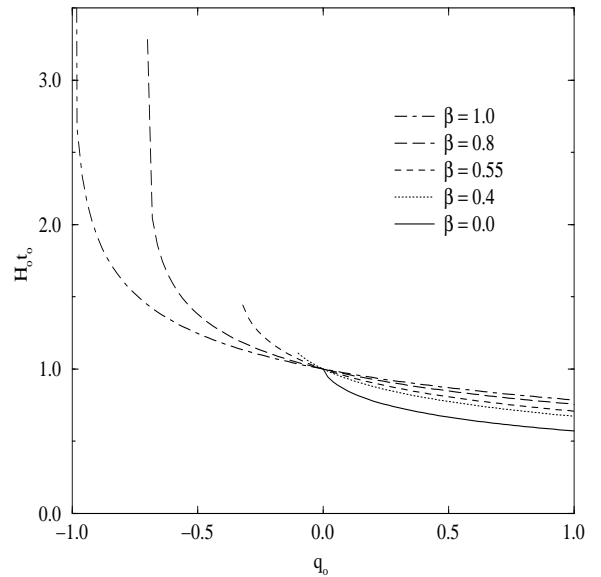


Fig. 2. Age parameter as a function of the deceleration parameter for some selected values of β . Solid curve is the standard FRW Universe with no matter creation ($\beta = 0, q_o \geq 0$). It follows from eq.(12) that for $\beta \geq 1/3$, the deceleration parameter assume negative values.

For $\Omega_o = 1$ these expressions reduce to the flat case studied in the paper I. Generically, we see that matter creation increases the dimensionless parameter $H_o t_o$ while preserving the overall expanding FRW behavior. The lookback time curves as a function of the redshift for some selected values of Ω_o and β are displayed in Fig. 1. For completeness, in Fig. 2 we show the age of the Universe (in units of H_o) as a function of the deceleration parameter.

b) Luminosity distance-redshift

The luminosity distance of a light source is defined as the ratio of the detected energy flux L , and the apparent luminosity, i.e., $d_l^2 = \frac{L}{4\pi l}$. In the standard FRW metric (1) it takes the form (Sandage 1988)

$$d_l = R_o r_1(z)(1+z) \quad , \quad (15)$$

where $r_1(z)$ is the radial coordinate distance of the object at light emission. Inserting $r_1(z)$ derived in the Appendix, it follows that

$$d_l = \frac{(1+z) \sin[\delta \sin^{-1}(\alpha_1) - \delta \sin^{-1}(\alpha_2)]}{H_o(\Omega_o - 1)^{\frac{1}{2}}} \quad . \quad (16)$$

where $\delta = \frac{2}{(1-3\beta)}$, $\alpha_1 = (\frac{\Omega_o - 1}{\Omega_o})^{\frac{1}{2}}$, and $\alpha_2 = \alpha_1(1+z)^{-\frac{1-3\beta}{2}}$.

As one may check, expressing Ω_o in terms of q_o from (10), and taking the limit $\beta \rightarrow 0$, the above expression reduces to

$$d_l = \frac{1}{H_o q_o^2} [z q_o + (q_o - 1)(\sqrt{2q_o z + 1} - 1)] \quad , \quad (17)$$

which is the usual FRW result (Weinberg 1972). Expanding eq.(14) for small z gives

$$H_o d_l = z + \frac{1}{2} \left(1 - \frac{1-3\beta}{2} \Omega_o\right) z^2 + \dots \quad , \quad (18)$$

which depends explicitly on the matter creation β parameter. However, replacing Ω_o from (10) we recover the usual FRW expansion for small redshifts, which depends only on the effective deceleration parameter q_o (Weinberg 1972). This is not a surprising result since expanding $d_l(z)$ in terms of Ω_o , the OO component of Einstein's equations has implicitly been considered, while the expansion in terms of q_o comes only from the form of the FRW line element. The luminosity distance as a function of the redshift for closed and open models with adiabatic matter creation is shown in Figures 3 and 4, respectively. As expected for all kinematic tests, different cosmological models have similar behavior at $z \ll 1$, and the greatest discrimination among them comes from observations at large redshifts.

c) Angular size-redshift

Another important kinematic test is the angular size - redshift relation ($\theta(z)$). As widely known, the data concerning the angular-size are until nowadays somewhat controversial (see Buchalter et al. 1998 and

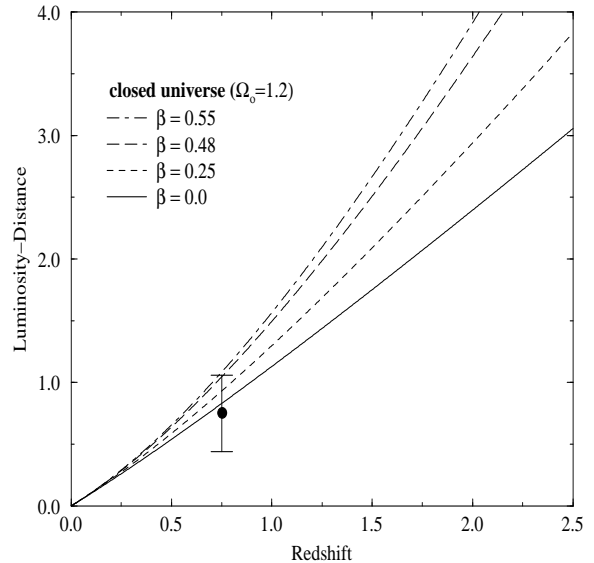


Fig. 3. Luminosity distance as a function of the redshift for closed models with adiabatic matter creation. Solid curve is the standard FRW closed model ($\beta = 0$). The selected values of β are shown in the picture. Here and in Fig. 4 the typical error bar and data point are taken from Kristian et al. (1978).

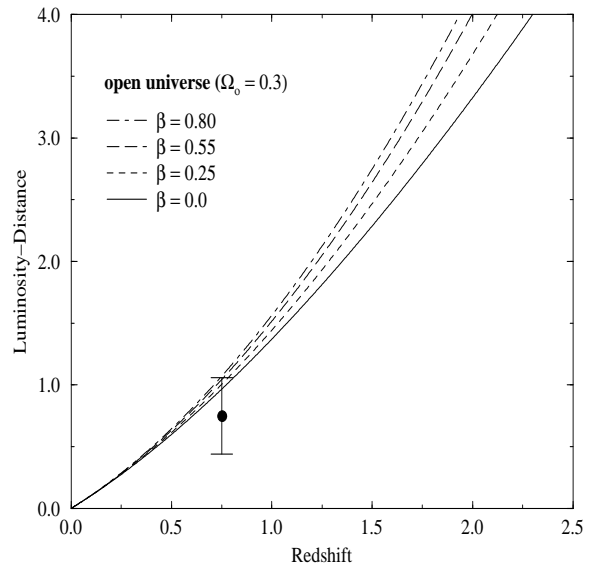


Fig. 4. The same graph of Fig. 3 for open models. Solid curve is the standard FRW open model ($\beta = 0$).

references therein). Here we are interested in angular diameters of light sources described as rigid rods and not as isophotal diameters. These quantities are naturally different, because in an expanding world the surface brightness varies with the distance (Sandage 1988). The angular size of a light source of proper size D (assumed free of evolutionary effects) located at $r = r_1(z)$ and observed at $r = 0$ is

$$\theta = \frac{D(1+z)}{R_o r_1(z)} . \quad (19)$$

Inserting the expression of $r_1(z)$ given in the Appendix it follows that

$$\theta = DH_o(\Omega_o - 1)^{\frac{1}{2}}(1+z) \sin[\delta \sin^{-1} \alpha_2 - \delta \sin^{-1} \alpha_1] . \quad (20)$$

For small z one finds

$$\theta = \frac{DH_o}{z} \left[1 + \frac{1}{2} \left(3 + \frac{1-3\beta}{2} \Omega_o \right) z + \dots \right] . \quad (21)$$

Hence, “adiabatic” matter creation as modelled here also requires an angular size decreasing as the inverse of the redshift for small z . However, for a given value of Ω_o , the second order term is a function only of the β parameter. In terms of q_o , inserting (10) into (19) it is readily obtained

$$\theta = \frac{DH_o}{z} \left[1 + \frac{1}{2} (3 + q_o) z + \dots \right] , \quad (22)$$

which is formally the same FRW result for small redshifts (Sandage 1988). At this limit only the effective deceleration parameter may be constrained from the data, or equivalently, at small redshifts one cannot extract the values of Ω_o and β separately. The angular size-redshift diagram for closed and open models and selected values of the β parameter is displayed in Figures 5 and 6, respectively.

d) Number counts-redshift

The final kinematic test considered here is the galaxy number count per redshift interval. We first notice that although modifying the evolution equation driving the amplification of small perturbations, and so the usual adiabatic treatment for galaxy formation, the created matter is smeared out and does not change the total number of sources present in the nonlinear regime. In other words, the number of galaxies already formed scales with R^{-3} (Lima et al. 1996, Lima and Alcaniz 1999).

The number of galaxies in a comoving volume is equal to the number density of galaxies (per comoving volume) n_g , times the comoving volume element dV_c

$$dN_g(z) = n_g dV_c = \frac{n_g r^2 dr d\Omega}{\sqrt{1 - kr^2}} . \quad (23)$$

By using $r_1(z)$ as derived in appendix, it follows that the general expression for number-counts can be written as

$$\frac{(H_o R_o)^3 dN_g}{n_g z^2 dz d\Omega} = \frac{\sin^2 \delta [\sin^{-1}(\alpha_2 - \sin^{-1}(\alpha_1))]}{(1+z) z^2 f(\Omega_o, \beta, z)} , \quad (24)$$

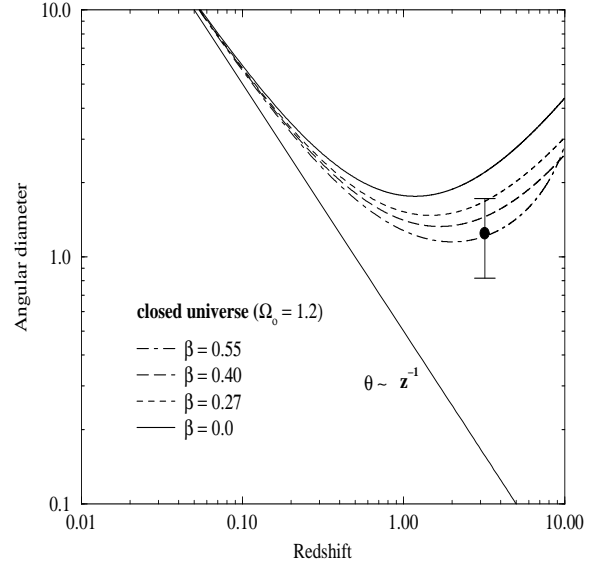


Fig. 5. Angular diameter versus redshift in closed models with adiabatic matter creation and some selected values of β . Solid curve is the standard model ($\beta = 0$). The angular size reaches a minimum at a given z_c and increases for fainter magnitudes. The minimum is displaced for higher z as the β parameter is increased. The straight line is the Euclidian result. Here and in Fig. 6, the typical error bar and data point are taken from Gurvits (1994).

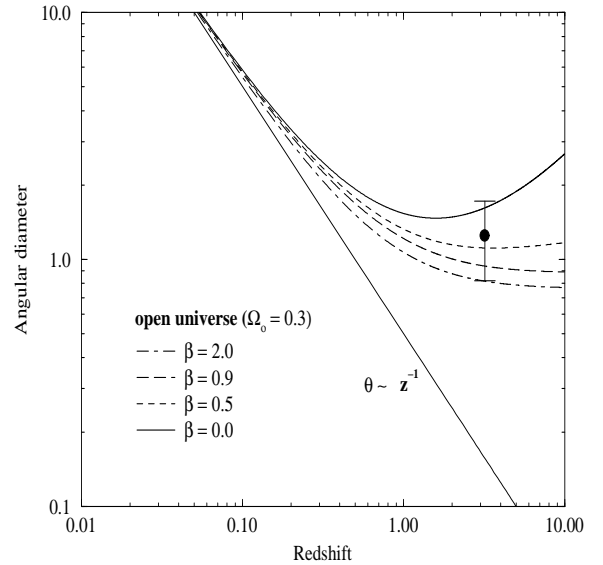


Fig. 6. Angular diameter versus redshift: the open case. As in Fig. 5, the straight line is the Euclidian result. Solid curve is the prediction of the standard FRW open universe ($\beta = 0$).

where $f(\Omega_o, \beta, z) = (\Omega_o - 1)[1 - \Omega_o + \Omega_o(1+z)^{1-3\beta}]^{1/2}$. For small redshifts

$$\frac{(H_o R_o)^3 dN_g}{n_g z^2 dz d\Omega} = 1 - 2\left(\frac{\Omega_o(1-3\beta)}{2} + 1\right)z + \dots \quad (25)$$

In Figures 7 and 8, we have displayed the number counts-redshift relations of closed and open Universes for some selected values of Ω_o and β . It is worth mentioning the tendency of matter creation models to have larger volumes per redshift interval than the standard FRW models with the same Ω_o . This feature is similar to the one found in decaying vacuum cosmologies and could turn out to be advantageous if the observational data indicate an excess count of high-redshift objects (Waga 1993). The limits on the β parameter obtained from all kinematic tests are shown in Table 1.

4. Conclusion

The recent observational evidences for an accelerated state of the present Universe, obtained from distant SNe Ia (Perlmutter et al. 1998) give a strong support to the search of alternative cosmologies. As demonstrated here, the process of adiabatic matter creation is also an ingredient accounting for this unexpected observational result. In a previous analysis (Lima and Alcaniz 1999) we have examined such a possibility for a flat Universe, while in the present paper we extend all the analysis for closed and open cosmologies. In this way, the expanding “postulate” and its main consequences may also be compatibilized with a cosmic fluid endowed with adiabatic matter creation.

The rather slight changes introduced by the matter creation process, which is quantified by the β parameter, provides a reasonable fit of several cosmological data. Kinematic tests like luminosity distance, angular diameter and number-counts versus redshift relations constrain perceptively the matter creation parameter. For models characterized by the pair (Ω_o, β) , the age of the Universe is always greather than the corresponding FRW model ($\beta = 0$), and even values bigger than H_o^{-1} are allowed for all values of the curvature parameter. However, in spite of these important physical consequences, the matter creation rate nowadays, $\psi_o = 3n_o H_o \approx 10^{-16}$ nucleons $cm^{-3} yr^{-1}$, is nearly the same rate predicted by the steady-state Universe (Hoyle et al. 1993) regardless the value of the curvature parameter. This matter creation rate is presently far below detectable limits.

A. Dimensionless radial coordinate versus redshift relation

Some observable quantities in the standard FRW model are easily determined expressing the radial dimensionless coordinate r of a source light as a function of the redshift (Mattig 1958). In this appendix, we derive a similar

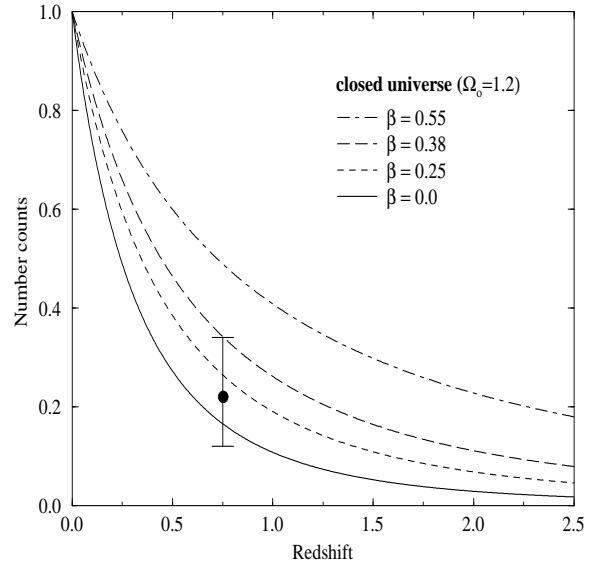


Fig. 7. Number counts as a function of the redshift for closed models with adiabatic matter creation. All results are shown for $\Omega_o = 1.5$ and some values of β . Solid curve is the closed FRW model with no matter creation. Here and in Fig. 8, the typical error bar and data point are taken from Loh and Spillar (1986).

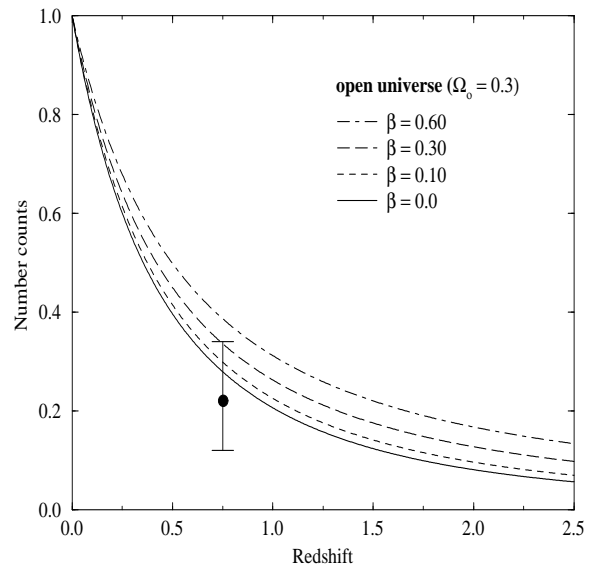


Fig. 8. Number counts versus redshift: the open case. As in Fig. 7, solid curve is the prediction of the standard FRW open universe ($\beta = 0$).

Table 1. Limits to β

Test	Open	Closed
Luminosity distance-redshift	$\beta \leq 0.55$	$\beta \leq 0.48$
Angular size-redshift	$\beta \leq 3.0$	$\beta \geq 0.27$
Number counts-redshift	$\beta \leq 0.30$	$\beta \leq 0.38$

equation to the matter creation scenario discussed in this paper.

Now consider a typical galaxy located at (r_1, θ_1, ϕ_1) emitting radiation to an observer at $(0, \theta_1, \phi_1)$. If the waves leave the source at time t_1 and reach the observer at time t_0 , the null geodesic equation ($dt^2 - \frac{R^2 dr^2}{1-kr^2} = 0$), which define the light track yields

$$\int_{t_0}^{t_1} \frac{dt}{R(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \frac{\text{Arcsin}\sqrt{kr_1}}{\sqrt{k}} = I \quad . \quad (\text{A1})$$

Since $t = t(R)$, changing variable to $x = \frac{R}{R_o}$ and using (10), the above result reads

$$I = \frac{1}{R_o H_o} \int_{(1+z)^{-1}}^1 [1 - \Omega_o + \Omega_o x^{-(1-3\beta)}]^{-1/2} \frac{dx}{x} \quad . \quad (\text{A2})$$

This integral depends on the values of the Ω_o and β parameters. For $\beta = \frac{1}{3}$ one finds the same results of the coasting cosmology (Kolb 1989). For β different of $\frac{1}{3}$, we introduce a new auxiliar variable $y^2 = (\frac{\Omega_o-1}{\Omega_o})x^{(1-3\beta)}$, in terms of which the above equation becomes

$$\frac{\text{Arcsin}\sqrt{kr_1}}{\sqrt{k}} = \frac{\delta}{R_o H_o (\Omega_o - 1)^{\frac{1}{2}}} \int_{\alpha_1}^{\alpha_2} \frac{dy}{\sqrt{1-y^2}} \quad , \quad (\text{A3})$$

where $\delta = \frac{2}{(1-3\beta)}$, $\alpha_2 = (\frac{\Omega_o-1}{\Omega_o})^{\frac{1}{2}}$, and $\alpha_1 = \alpha_2(1+z)^{-\frac{(1-3\beta)}{2}}$.

The right hand side of the above integral is the same appearing in (A1) for $k = 1$. Hence, replacing in (A3) the value of k given by (2) and (9), it is readily seen that

$$r_1(z) = \frac{\sin[\delta \sin^{-1} \alpha_2 - \delta \sin^{-1} \alpha_1]}{R_o H_o (\Omega_o - 1)^{\frac{1}{2}}} \quad . \quad (\text{A4})$$

In particular, the limit for a flat Universe ($\Omega_o = 1$) yields (paper I)

$$r_1(z) = \frac{2}{(1-3\beta)R_o H_o} \{1 - (1+z)^{\frac{2}{1-3\beta}}\} \quad , \quad (\text{A5})$$

which could have been obtained directly from (A2).

In terms of the deceleration parameter (A4) may be rewritten as

$$r_1(z) = \frac{\sin[\delta \sin^{-1} \alpha_2 - \delta \sin^{-1} \alpha_1]}{R_o H_o (\frac{2q_o}{1-3\beta} - 1)^{\frac{1}{2}}} \quad , \quad (\text{A6})$$

which in the limit $\beta \rightarrow 0$ reduces to the usual FRW result (Weinberg 1972)

$$r_1(z) = \frac{q_o z + (q_o - 1)(\sqrt{2q_o z + 1} - 1)}{H_o R_o q_o^2 (1+z)} \quad . \quad (\text{A7})$$

Equation (A4), or equivalently (A6), plays a key role in the derivation of some astrophysical quantities discussed in this paper.

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References

- Alcaniz J. S. and Lima J. A. S.: 1999, Ap.J Lett., in press (astro-ph/9902298)
- Bolte M., Hogan C.J.: 1995, Nature 376, 399
- Buchalter et al.: 1998, Ap. J., 222, 111
- Caldwell R.R., Dave R., Steinhardt P.J.: 1998, Phys. Rev. Lett. 80, 1582
- Calvão M.O., Lima J.A.S., Waga I.: 1992, Phys. Lett. A 162, 233
- Carvalho J.C., Lima J.A.S., Waga I.: 1992, Phys. Rev. D46, 2404
- Chiba T. et al.: 1997, MNRAS, 289, 5
- Dunlop, J. et al. 1996, Nature, 381, 581
- Dunlop, J. et al. 1998, astro-ph/9801114
- Freedman W.L.: 1998 in Proceedings of the 18th Texas Symposium on Relativistic Astrophysics, edited by A. Olinto, J. Frieman, and D. Schramm, World Scientific
- Gurvits L.I.: 1994, Ap. J. 425, 442
- Hoyle F., Burbidge G., Narlikar J.V.: 1993, Ap.J., 410, 437
- Huey G. et al.: (1998) Phys. Rev D57, 2152
- Kolb E.W.: 1989, Ap. J., 344, 543
- Kolb E.W.: and Turner M.S.: 1990, The Early Universe, Redwood City: Addison-Wesley
- Krauss, L.M.: 1997, Ap. J., 480, 486
- Krauss, L.M.: 1998, Ap. J., 501, 461
- Krauss, L.M.: and Turner, M. 1995, Gen. Rel. Grav., 27, 1137
- Lima J.A.S., Germano A.S., Abramo L.R.W.: 1996, Phys. Rev. D53, 4287
- Lima J.A.S. and Alcaniz J.S.: 1999, Astron. and Astrop. In press (astro-ph/9902337)
- Lima J.A.S. and Abramo R.: 1999, Phys. Lett. A, in press
- Mattig W.: 1958, Astron. Nachr. 284, 109
- Loh E., Spillar E.: 1986, Ap. J. 307, L1
- Prigogine I. et al.: 1989, Gen. Rel. Grav. 21, 767
- Perlmutter S. et al.: 1998, Nature 391, 51
- Perlmutter S. et al.: astro-ph/9812113
- Pont F. et al.: 1998, Astron. and Astrop. 329, 87
- Ratra B. and Peebles P.J.E.: 1988, Phys. Rev. D37, 3407
- Riess A. G. et al.: 1998, AJ, 116, 1009

- Sandage A.: 1988, *Ann. Rev. Astron. Astrophys.* 26, 561
Turner M.S. and White M.: 1997, *Phys. Rev. D* 56, R4439
Waga I.: 1993, *Ap.J.*, 414, 436
Weinberg S.: 1972, *Cosmology and Gravitation*, John Wiley
Sons, N.Y.